

# Magnetic moments of negative-parity baryons in QCD

T. M. Aliev <sup>\*†</sup>, M. Savcı <sup>‡</sup>

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

## Abstract

Using the most general form of the interpolating current for the octet baryons, the magnetic moments of the negative-parity baryons are calculated within the light-cone sum rules. The contributions coming from diagonal transitions of the positive-parity baryons, and also from non-diagonal transition between positive and negative-parity baryons are eliminated by considering the combinations of different sum rules corresponding to the different Lorentz structures. A comparison of our results on magnetic moments of the negative-parity baryons with the other approaches existing in literature is presented.

PACS numbers: 11.55.Hx, 13.40.Gp, 14.20.Gk

---

<sup>\*</sup>taliev@metu.edu.tr

<sup>†</sup>Permanent address: Institute of Physics, Baku, Azerbaijan.

<sup>‡</sup>savci@metu.edu.tr

# 1 Introduction

The study of the spectroscopy, properties and structure of hadrons plays a critical role in understanding the strong interaction at low energies. Study of the physics of negative-parity baryons in this direction receives special attention. There is yet very limited experimental information about negative-parity hyperons. Comprehensive studies on theoretical and experimental sides can shed light in understanding dynamics of the negative-parity baryons. For example, the mass difference of the positive and negative-parity baryons can be attributed to the spontaneous breaking of the chiral symmetry. In QCD sum rules approach [1] the spontaneous breakdown of the chiral symmetry is related with the condensates of chiral-odd operators (see for example [2]).

The measurement of the magnetic moment of the negative-parity baryons can give useful information about their inner structure. Some number of photo and electro-production experiments are planned to measure the magnetic moments of these baryons at Mainz Microton facility [3, 4], and Jefferson Laboratory [5]. The magnetic moments of negative-parity baryons have already been calculated in framework of the chiral and non-relativistic constituent quark models [6], lattice QCD [7], simple quark model [8], unitarized chiral perturbation theory [9], and effective Hamiltonian approach [10]. However, there are drastic differences among the predictions of the above-mentioned approaches. Therefore, motivated partly by the experimental studies, there appears the necessity to calculate the magnetic moments of these baryons in framework of the approaches other than listed above. In the present work we calculate the magnetic moment of the negative-parity baryons in framework of the light-cone QCD sum rules. The main advantage of the QCD sum rules compared to the other approaches is that it is based on fundamental QCD Lagrangian, and it takes into account the non-perturbative nature of the QCD vacuum. The light-cone version of the QCD sum rules is based on the operator product expansion (OPE) near the light-cone. This expansion is performed over the twists of the operator rather than the dimension of the operators, as is the case in the traditional QCD sum rules. The non-perturbative dynamics is described by the light-cone distribution amplitudes, which appear in the matrix elements of the nonlocal operators between the vacuum and corresponding one-particle states. This method have successfully applied to wide range of problems in hadron physics (for more about this method, see [11]).

The structure of the present work as follows. In section 2 we formulate and derive the light-cone sum rules for the magnetic moments of the negative-parity octet baryons. In the same section we also obtain the sum rules for the masses and residues of these baryons. In section 3 we present our numerical results for the magnetic moment of the negative-parity baryons, and discussion and conclusion on the obtained results.

## 2 Sum rules for the magnetic moments of negative-parity baryons

In order to obtain light-cone QCD sum rules for the magnetic moments of the negative-parity baryons, we start by considering the following correlation function:

$$\Pi = i \int d^4x e^{ipx} \langle 0 | T \{ \eta_B(x) \bar{\eta}_B(0) \} | 0 \rangle_\gamma, \quad (1)$$

where  $\eta_B$  is the interpolating current of the corresponding baryon. According to the QCD sum rules methodology, in order to construct sum rules for the appropriate physical quantity, the correlation function is calculated in terms of hadrons and quark-gluon degrees of freedom, and then with the help of quark-hadron duality ansatz these two representations are related by using the analytical continuation.

In calculating the correlation function from QCD side, we need the expressions of the interpolating currents of the baryons. The general form of the interpolating currents of octet baryons are given in [12] (see also [13]).

Let us now obtain the phenomenological representation of the correlator function. Saturating (1) with the complete set of baryons having the same quantum numbers as the interpolating current and isolating the contributions of the ground state, we get the following expression for the correlator function:

$$\begin{aligned} \Pi = & \frac{\langle 0 | \eta_B | B(p_2) \rangle}{(p_2^2 - m_B^2)} \langle B(p_2) \gamma(q) | B(p_1) \rangle \frac{\langle B(p_1) | \bar{\eta}_B | 0 \rangle}{(p_1^2 - m_B^2)} \\ & + \frac{\langle 0 | \eta_B | B^*(p_2) \rangle}{(p_2^2 - m_B^{*2})} \langle B^*(p_2) \gamma(q) | B^*(p_1) \rangle \frac{\langle B^*(p_1) | \bar{\eta}_B | 0 \rangle}{(p_1^2 - m_B^{*2})} \\ & + \frac{\langle 0 | \eta_B | B(p_2) \rangle}{(p_2^2 - m_B^2)} \langle B(p_2) \gamma(q) | B^*(p_1) \rangle \frac{\langle B^*(p_1) | \bar{\eta}_B | 0 \rangle}{(p_1^2 - m_B^{*2})} \\ & + \frac{\langle 0 | \eta_B | B^*(p_2) \rangle}{(p_2^2 - m_B^{*2})} \langle B^*(p_2) \gamma(q) | B(p_1) \rangle \frac{\langle B(p_1) | \bar{\eta}_B | 0 \rangle}{(p_1^2 - m_B^2)} \\ & + \dots \end{aligned} \quad (2)$$

where dots correspond to the contribution of the higher states and continuum; and  $B^*$  is the negative-parity baryon.

The matrix elements entering into Eq. (2) are defined as

$$\begin{aligned} \langle 0 | \eta | B(p) \rangle &= \lambda_B u(p), \\ \langle 0 | \eta | B^*(p) \rangle &= \lambda_{B^*} \gamma_5 u(p), \\ \langle B(p_2) \gamma(q) | B(p_1) \rangle &= e \varepsilon^\mu \bar{u}(p_2) \left[ f_1 \gamma_\mu - \frac{i \sigma_{\mu\nu} q^\nu}{2m_B} f_2 \right] u(p_1), \\ \langle B^*(p_2) \gamma(q) | B^*(p_1) \rangle &= e \varepsilon^\mu \bar{u}(p_2) \left[ f_1^* \gamma_\mu - \frac{i \sigma_{\mu\nu} q^\nu}{2m_B^*} f_2^* \right] u(p_1), \\ \langle B^*(p_2) \gamma(q) | B(p_1) \rangle &= e \varepsilon^\mu \bar{u}(p_2) \left[ f_1^T \gamma_\mu - \frac{i \sigma_{\mu\nu} q^\nu}{m_B + m_B^*} f_2^T \right] \gamma_5 u(p_1). \end{aligned} \quad (3)$$

The structure proportional to  $\gamma_\mu$  for the diagonal transformations contains the factor  $f_1 + f_2$ , and at  $q^2 = 0$  it corresponds to the magnetic moments of the corresponding baryons. Performing summation over the spins of the baryons for the terms proportional to  $\gamma_\mu$  we get

$$A'(\not{p}_2 + m_B)\not{\epsilon}(\not{p}_1 + m_B) + C'(\not{p}_2 - m_{B^*})\not{\epsilon}(\not{p}_1 - m_{B^*}) + D'(\not{p}_2 - m_{B^*})\not{\epsilon}(\not{p}_1 + m_B) + D'(\not{p}_2 + m_B)\not{\epsilon}(\not{p}_1 - m_{B^*}) , \quad (4)$$

where

$$\begin{aligned} A' &= \frac{\lambda_B(t')\lambda_B(t)}{(m_B^2 - p_2^2)(m_B^2 - p_1^2)}(f_1 + f_2) , \\ C' &= \frac{\lambda_{B^*}(t')\lambda_{B^*}(t)}{(m_{B^*}^2 - p_2^2)(m_{B^*}^2 - p_1^2)}(f_1^* + f_2^*) , \\ D' &= \frac{\lambda_B(t')\lambda_{B^*}(t)}{(m_{B^*}^2 - p_2^2)(m_B^2 - p_1^2)}\left(f_1^T + \frac{m_{B^*} - m_B}{m_{B^*} + m_B}f_2^T\right) , \\ E' &= -\frac{\lambda_{B^*}(t')\lambda_B(t)}{(m_B^2 - p_2^2)(m_{B^*}^2 - p_1^2)}\left(f_1^T + \frac{m_{B^*} - m_B}{m_{B^*} + m_B}f_2^T\right) , \end{aligned} \quad (5)$$

where  $t$  and  $t'$  are two arbitrary parameters in interpolating currents of the baryons. In order to determine the magnetic moments of the negative-parity baryons, the contributions coming from the diagonal  $B \rightarrow B$  and non-diagonal  $B \rightarrow B^*$  and  $B^* \rightarrow B$  transitions should be removed, i.e., only the terms proportional to  $C'$  need to be determined. In the case of diagonal transitions the quantities  $(f_1 + f_2)|_{q^2=0}$  and  $(f_1^* + f_2^*)|_{q^2=0}$  correspond to the magnetic moments of the positive and negative octet baryons in natural units. As a result we have four equations in determining the magnetic moment of the negative-parity baryons.

As has already been noted, in constructing the sum rules for the magnetic moments the expression for the correlation function from the QCD side is also needed. As an example we shall present the result for the  $\Sigma^{*+}$  case:

$$\begin{aligned} \Pi^{\Sigma^+} &= 4\epsilon_{abc}\epsilon_{a'b'c'} \int d^4x e^{ipx} \langle \gamma(q) | \sum_{\ell=1}^2 \sum_{k=1}^2 \left\{ \Gamma_2^{\ell} S_u^{cc'}(x) \Gamma_2^k \text{Tr} \left( S_s^{bb'}(x) C \Gamma_1^k S_u^{aa'T}(x) C \Gamma_1^\ell \right) \right. \\ &\quad + \Gamma_2^\ell \left[ S_u^{cc'}(x) (C \Gamma_1^k)^T S_s^{bb'}(x) (C \Gamma_1^\ell)^T S_u^{aa'}(x) \Gamma_2^k + S_u^{aa'}(x) (C \Gamma_1^k)^T S_s^{bb'T}(x) (C \Gamma_1^\ell)^T S_u^{cc'}(x) \Gamma_2^k \right. \\ &\quad \left. \left. + S_u^{aa'}(x) \Gamma_2^k \text{Tr} \left( S_s^{bb'T}(x) C \Gamma_1^k S_u^{cc'T}(x) C \Gamma_1^\ell \right) \right] \right\} | 0 \rangle , \end{aligned} \quad (6)$$

where  $a, b, c, a', b', c'$  are the color indices;  $S_q$  is the light quark propagator. The results for the  $\Sigma^0$ ,  $\Sigma^-$ ,  $\Xi^0$  and  $\Xi^-$  baryons can easily be obtained from the above expression of the correlation function for  $\Sigma^+$  with the help of the following replacements:

$$\begin{aligned} \Pi^{\Sigma^-} &= \Pi^{\Sigma^+}(u \rightarrow d) , \\ \Pi^{\Sigma^0} &= \frac{1}{2}(\Pi^{\Sigma^+} + \Pi^{\Sigma^-}) \\ \Pi^{\Xi^0} &= \Pi^{\Sigma^+}(u \rightarrow s) , \\ \Pi^{\Xi^-} &= \Pi^{\Xi^0}(u \rightarrow d) . \end{aligned} \quad (7)$$

The expression for the light-quark operator we use in further analysis has the following form (see for example [13]):

$$\begin{aligned}
S_q(x) &= \langle 0 | T \{ \bar{q}(x) q(0) \} | 0 \rangle \\
&= \frac{i \not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left( 1 - \frac{im_q}{4} \not{x} \right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left( 1 - \frac{im_q}{6} \not{x} \right) \\
&\quad - ig_s \int_0^1 dv \left[ \frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(vx) \sigma^{\mu\nu} - vx^\mu G_{\mu\nu}(vx) \gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
&\quad \left. - \frac{im_q}{32\pi^2} G_{\mu\nu} \sigma^{\mu\nu} \left( \ln \frac{-x^2 \Lambda^2}{4} + 2\gamma_E \right) \right], \tag{8}
\end{aligned}$$

where  $\Lambda$  is the energy cut off separating perturbative and non-perturbative sectors. We keep only linear quark mass terms in (8) since the contributions coming from higher order mass terms are negligibly small. It can also easily be seen from the expression of the light-quark propagator that, one-gluon operator is retained while the two-gluon  $\bar{q}GGq$  and four-quark  $\bar{q}q\bar{q}q$  operators are neglected. Neglecting these operators is justified with the help of the conformal spin expansions [14, 15].

Few words about the calculation technique are in order. If a photon interact with the quark fields perturbatively, its contribution can be calculated by replacing one of the propagators in Eq. (6) with the following one:

$$S = -\frac{1}{2} \int dy y_\nu \mathcal{F}^{\mu\nu} S^{free}(x-y) \gamma_\mu S^{free}(y), \tag{9}$$

where  $\mathcal{F}^{\mu\nu}$  is the electromagnetic field strength tensor, and the Fock-Schwinger gauge  $x_\mu A^\mu = 0$  has already been taken into account in the relation  $A_\mu = \frac{1}{2} \mathcal{F}_{\mu\nu} y^\nu$ . Note also that,  $S^{free}$  in Eq. (9) is the free quark operator which corresponds to the first term on the right hand side of Eq. (8), and the following two propagators are the full propagators of the quarks. The contribution coming from the photon's interacting with a quark field non-perturbatively is described by replacing one of the propagators with

$$S_{\rho\sigma}^{ab} = -\frac{1}{4} \bar{q}^a \Gamma_j q^b (\Gamma_j)_{\rho\sigma},$$

where  $\Gamma_j$  describes the full set of Dirac matrices; and the two remaining propagators are described by Eq. (8).

Substituting Eq. (9) into (6) we see that the contributions corresponding to the non-perturbative interaction of a photon with the quark field appear in the matrix elements of the nonlocal operators  $\bar{q}\Gamma_j q$ ,  $\bar{q}G_{\mu\nu}\Gamma_j q$  between the vacuum and photon states, i.e.,  $\langle \gamma | \bar{q}\Gamma_j q | 0 \rangle$  and  $\langle \gamma | \bar{q}G_{\mu\nu}\Gamma_j q | 0 \rangle$ , respectively. These matrix elements are determined in terms of the photon distribution amplitudes (DAs) which are given in [16].

Using Eqs. (6) and (8), and the definitions of DAs of a photon, one can calculate the theoretical part of the correlation function. We present the explicit expressions of invariant functions  $\Pi_1$ ,  $\Pi_2$ ,  $\Pi_3$  and  $\Pi_4$  corresponding to the structures  $\not{p}\not{q}$ ,  $\not{p}\not{x}$ ,  $\not{x}\not{q}$  and  $\not{x}$  for the  $\Sigma^{0*}$  baryon, respectively. The results for the  $\Lambda^*$  baryon can be obtained by using the following relation between the interpolating currents of  $\Sigma^{0*}$  and  $\Lambda^*$  baryons [17]:

$$2\eta_{\Sigma^0}(d \leftrightarrow s) = -\eta_{\Sigma^0} - \sqrt{3}\eta_\Lambda.$$

The final step in deriving the sum rules for the magnetic moment of the negative-parity baryons is performing double Borel transformation on the variables  $p_1^2 = (p+q)^2$  and  $p_2^2 = p^2$  on both sides of the correlation function and equating the coefficients of the corresponding structures, the result of which is

$$\begin{aligned} A + C + D + E &= \Pi_1^{Bor} , \\ m_B A - m_{B^*} C - m_{B^*} D + m_B E &= \Pi_2^{Bor} , \\ (m_B A + m_{B^*})(D - E) &= \Pi_3^{Bor} , \\ m_B^2 A + m_{B^*}^2 C - m_B m_{B^*}(D + E) &= \Pi_4^{Bor} , \end{aligned} \tag{10}$$

where

$$\begin{aligned} A &= \lambda_B(t') \lambda_B(t) e^{-\frac{m_B^2}{M_1^2} - \frac{m_B^2}{M_2^2}} (f_1 + f_2) , \\ C &= \lambda_{B^*}(t') \lambda_{B^*}(t) e^{-\frac{m_{B^*}^2}{M_1^2} - \frac{m_{B^*}^2}{M_2^2}} (f_1^* + f_2^*) , \\ D &= \lambda_{B^*}(t') \lambda_B(t) e^{-\frac{m_B^2}{M_1^2} - \frac{m_{B^*}^2}{M_2^2}} \left( f_1^T + \frac{m_{B^*} - m_B}{m_{B^*} + m_B} f_2^T \right) , \\ E &= -\lambda_B(t') \lambda_{B^*}(t) e^{-\frac{m_{B^*}^2}{M_1^2} - \frac{m_B^2}{M_2^2}} \left( f_1^T + \frac{m_{B^*} - m_B}{m_{B^*} + m_B} f_2^T \right) . \end{aligned}$$

The sum rules for the magnetic moments of the negative-parity baryons is obtained by solving Eq. (10) for  $C$  in terms of the invariant functions  $\Pi_i$ . Since  $C$  corresponds to the diagonal transition, we set  $t' = t$  in the expressions of the interpolating currents. The resulting expression of the magnetic moment of the negative-parity baryons is

$$\begin{aligned} \mu_{B^*} &= \frac{e^{m_{B^*}^2/M^2}}{\lambda_{B^*}^2(m_B + m_{B^*})(m_B^2 + 3m_{B^*}^2)} \left\{ \left[ m_B(m_B - m_{B^*}) - 2m_{B^*}^2 \right] \Pi_1^{Bor} \right. \\ &\quad \left. - 2m_B(m_B + m_{B^*}) \Pi_2^{Bor} - (m_B - 3m_{B^*}) \Pi_3^{Bor} - m_B(m_B + m_{B^*}) \Pi_4^{Bor} \right\} , \end{aligned} \tag{11}$$

where we have used  $M_1^2 = 2M^2$ ,  $M_2^2 = 2M^2$ . The expressions of  $\Pi_i^{Bor}$  are rather lengthy, and therefore we do not present them in the present work.

It follows from Eq. (11) that in determining numerical values of the magnetic moments of the negative-parity baryons the corresponding residues are needed. These residues can be obtained from the following correlator:

$$\Pi(p^2) = i \int d^4x e^{ipx} \left\langle 0 \left| T \left\{ \eta_B(x) \bar{\eta}_B(0) \right\} \right| 0 \right\rangle , \tag{12}$$

which can be written in terms of the invariant functions as

$$\Pi(p^2) = \Pi_1 \not{p} + \Pi_2 I .$$

Saturating (12) with the corresponding positive and negative baryons, we get

$$\Pi(p^2) = \frac{|\lambda_{B^*}|^2}{m_{B^*}^2 - p^2} (\not{p} - m_{B^*}) + \frac{|\lambda_B|^2}{m_B^2 - p^2} (\not{p} + m_B) . \tag{13}$$

By eliminating the contributions of the positive-parity baryon from these equations and performing Borel transformation over the variable  $p^2$ , for the mass and residues, we get

$$m_{B^*}^2 = \frac{\int_{s_0}^s s ds e^{-s/M^2} [m_B \text{Im}\Pi_1(s) - \text{Im}\Pi_2(s)]}{\int_{s_0}^s ds e^{-s/M^2} [m_B \text{Im}\Pi_1(s) - \text{Im}\Pi_2(s)]}$$

$$|\lambda_{B^*}|^2 = \frac{e^{m_{B^*}^2/M^2}}{m_B + m_{B^*}} \frac{1}{\pi} \int ds e^{-s/M^2} [m_B \text{Im}\Pi_1(s) - \text{Im}\Pi_2(s)] .$$

The spectral densities  $\text{Im}\Pi_1(s)$  and  $\text{Im}\Pi_2(s)$  are calculated in [13], and for this reason we do not present them in this work.

### 3 Numerical analysis

In this section we perform numerical calculations of the sum rules for magnetic moments of the negative-parity baryons. The main non-perturbative input parameters of the light-cone QCD sum rules for the magnetic moments are the photon DAs which are all given in [16].

The values of the other input parameters are  $\langle \bar{u}u \rangle (1 \text{ GeV}) = \langle \bar{d}d \rangle (1 \text{ GeV}) = -(0.243)^3 \text{ GeV}^3$ ,  $\langle \bar{s}s \rangle (1 \text{ GeV}) = 0.8 \langle \bar{u}u \rangle (1 \text{ GeV})$ ,  $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$  [18],  $\Lambda = (0.5 \pm 1.0) \text{ GeV}$  [19], and  $f_{3\gamma} = -0.039$  [16], the magnetic susceptibility  $\chi (1 \text{ GeV}) = -2.85 \pm 0.5 \text{ GeV}^{-2}$  [20] and  $m_s (2 \text{ GeV}) = 111 \pm 6 \text{ MeV}$  [21].

The sum rules for the magnetic moments of the negative-parity baryons contain three auxiliary parameters, namely, the arbitrary parameter  $t$  appearing in interpolating current, Borel mass parameter  $M^2$  and the continuum threshold  $s_0$ . Obviously the magnetic moments are expected to be independent of these parameters. In implementing the numerical analysis program, we first look for the region where magnetic moments of the negative-parity baryons are independent of  $M^2$  at properly chosen values of the parameters  $s_0$  and  $t$ . It should be noted here that the QCD sum rules analysis restricts the arbitrary parameter  $t$  to possess only positive values. The upper bound of  $M^2$  is obtained by demanding that the continuum contribution should be less than, say, half of the perturbative contributions. The lower bound of  $M^2$  is determined by requiring that the contributions of the highest power of  $1/M^2$  terms contribute less than 30% of the highest  $M^2$  terms. These two conditions lead to the following domains:

$$1.5 \leq M^2 \leq 2.5 \text{ GeV}^2 \text{ for } p^* \text{ and } n^*, \text{ and}$$

$$1.6 \leq M^2 \leq 3.0 \text{ GeV}^2 \text{ for } \Lambda^*, \Sigma^* \text{ and } \Xi^* .$$

For this reason as we draw figures depicting the  $M^2$  dependence of the magnetic moments of the negative-parity baryons positive values of the arbitrary parameter  $t$  have been used, and different “working regions” of  $M^2$  are used for different members of the negative-parity octet baryons. In these preferred regions of  $M^2$  magnetic moments of the negative-parity baryons are practically independent of  $M^2$  at fixed values of  $s_0$  and  $t$ .

The next problem which needs to be solved is finding the working region for the arbitrary parameter  $t$  where again magnetic moments of the negative-parity baryons are independent

of it. For this goal the mass sum rules have been used. We see that  $\lambda_{B^*}^2$  is almost independent of  $t$  if it varies in the region  $0.2 \leq t \leq 1.5$ , which is common for all negative parity baryons at fixed values of  $M^2$ . We shall use this boundary for  $t$  in further numerical analysis of the magnetic moment of the negative parity baryons. It should be noted here that when parameter  $t$  varies in this domain the mass sum rules predict the following values for the mass of the negative-parity baryons:  $m_\Lambda = 1.75 \text{ GeV}$ ,  $m_\Xi = 1.80 \text{ GeV}$ ,  $m_\Sigma = 1.7 \text{ GeV}$ , at  $s_0 = 4.0 \text{ GeV}^2$  and when the Borel parameter varies in the region  $1.4 \leq M^2 \leq 2.2 \text{ GeV}^2$ . These predictions for mass of the negative parity baryons are very close to their experimental values [22]. It should be stressed here that the Borel mass parameters appearing in the sum rules for magnetic moments and in the mass sum rules are different in general.

The final step of our procedure is determination of the region for the arbitrary parameter  $t$  by using the domain which follows from the mass sum rules analysis in which the sum rules exhibit good stability to the variation of  $t$  at fixed values of  $M^2$ . Our numerical calculations show that the regions of  $t$ , where the magnetic moments of the negative-parity baryons are independent of it, gets narrower. For example, the best stability of the magnetic moments are achieved when  $t$  lies in the region  $0.9 \leq t \leq 1.0$  for  $n^*$  and  $p^*$ , and  $0.6 \leq t \leq 0.7$  for the  $\Lambda^*$ ,  $\Sigma^*$  and  $\Xi^*$  baryons, respectively.

The results of our numerical analysis for the negative parity baryons are all summarized in Table I. For completeness, in Table I, we also present the predictions of the chiral and non-relativistic constituent quark models [6], lattice calculations [7], simple quark model [8] and unitarized chiral perturbation theory [9], and effective Hamiltonian approach [10]. It should be noted here that the magnetic moments of the  $\Lambda(1670)$  baryon is calculated within the framework of the unitarized chiral perturbation theory in [23] which predicts  $\mu_{\Lambda^*} = -0.29\mu_N$ , where  $\mu_N$  is the nuclear magneton.

From this table we see that the results obtained by the above-mentioned approaches are quite different, not only in magnitude, but also in sign in many cases. On the other hand, our results predicts that  $\mu_{\Xi^{*-}} \simeq \mu_{\Sigma^{*-}}$ ,  $\mu_{\Xi^{*0}} \simeq \mu_{n^*}$ ,  $\mu_{\Sigma^{*+}} \simeq \mu_{p^*}$ ,  $\mu_{\Sigma^{*-}} + \mu_{n^*} \simeq -\mu_{p^*}$ ,  $2\mu_{\Lambda^*} \simeq \mu_{n^*}$ , which are all very close to the exact  $SU(3)$  symmetry relation.

Naively, one can expect that the magnetic moments of the negative-parity baryons can be obtained from the positive ones by the relation

$$|\mu_-| = \frac{m_B}{m_{B^*}} |\mu_+| .$$

When used, this relation leads to the following results (for magnetic moments of the positive-parity baryons we have used their experimental results):  $|\mu_{p^*}| \simeq 1.7\mu_N$ ,  $|\mu_{n^*}| \simeq 1.1\mu_N$ ,  $\mu_{\Sigma^{*+}} \simeq 1.7\mu_N$ ,  $|\mu_{\Sigma^{*-}}| \simeq 0.8\mu_N$ ,  $|\mu_{\Sigma^{*0}}| \simeq 0.46\mu_N$ ,  $|\mu_{\Xi^{*0}}| \simeq \mu_N$ ,  $|\mu_{\Xi^{*-}}| \simeq 0.5\mu_N$ , and  $\mu_{\Lambda^*} \simeq -0.4\mu_N$ . When compared with these naive estimations we see that, except for the  $n^*$  case, our results are very close with the above-presented naive estimations.

In summary, using the light-cone QCD sum rules method we calculate the magnetic moments of the negative-parity octet baryons. In our analysis the contaminations originating from the diagonal transitions of the positive parity baryons, as well as non-diagonal transitions among negative and positive-parity baryons are all eliminated by taking the contributions coming from the sum rules corresponding to the different Lorentz structures into account. Furthermore, we present a comparison of our results on the magnetic moments of



the negative-parity octet baryons with those lattice, constituent quark model, unitarized chiral perturbation theory, and chiral constituent quark model predictions. It is observed that the predictions of these approaches totally differ from each other in many cases. Therefore, any future experimental measurement would be very important for choosing the “right” theory and understanding the structure of these baryons.

|                     | Our result | [6]    | [6]    | [7]  | [8]  | [9]   | [10]  |
|---------------------|------------|--------|--------|------|------|-------|-------|
| $\mu_{p^*}$         | 1.4        | 2.085  | 1.894  | -1.8 | 1.9  | 1.1   | 1.24  |
| $\mu_{n^*}$         | -0.54      | -1.569 | -1.284 | -1   | 1.2  | -0.25 | -0.84 |
| $\mu_{\Sigma^{+*}}$ | 1.8        | 1.8    | 1.814  | -0.6 | ...  | ...   | ...   |
| $\mu_{\Sigma^{0*}}$ | 0.4        | 0.79   | 0.82   | 0.1  | ...  | ...   | ...   |
| $\mu_{\Sigma^{-*}}$ | -1.1       | -1     | -0.689 | 1    | ...  | ...   | ...   |
| $\mu_{\Xi^{0*}}$    | -0.55      | -1.442 | -0.99  | -0.5 | ...  | ...   | ...   |
| $\mu_{\Xi^{-*}}$    | -1.2       | -0.165 | -0.315 | -0.8 | ...  | ...   | ...   |
| $\mu_{\Lambda^*}$   | -0.26      | ...    | ...    | -0.1 | -1.9 | -0.29 | ...   |

Table 1: The magnetic moments of the negative-parity octet baryons in units of nuclear magneton  $\mu_N$  predicted by the light cone QCD sum rules (our result), chiral and non-relativistic constituent quark models [6], lattice calculations [7], simple quark model [8], unitarized chiral perturbation theory [9], non-relativistic constituent quark model [6], and effective Hamiltonian [10] approaches.

## Acknowledgments

We thank V. S. Zamiralov for his collaboration at the early stage of this work and K. Azizi for useful discussions.

## Appendix A

For completeness, in this Appendix we present the matrix elements  $\langle \gamma(q) | \bar{q}\Gamma_i q | 0 \rangle$  which are calculated in terms of the photon distribution amplitudes (DAs) [10].

$$\begin{aligned}
\langle \gamma(q) | \bar{q}(x) \sigma_{\mu\nu} q(0) | 0 \rangle &= -ie_q \bar{q}q (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left( \chi \varphi_\gamma(u) + \frac{x^2}{16} \mathbb{A}(u) \right) \\
&\quad - \frac{i}{2(qx)} e_q \langle \bar{q}q \rangle \left[ x_\nu \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) - x_\mu \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu q(0) | 0 \rangle &= e_q f_{3\gamma} \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi^v(u) \\
\langle \gamma(q) | \bar{q}(x) \gamma_\mu \gamma_5 q(0) | 0 \rangle &= -\frac{1}{4} e_q f_{3\gamma} \epsilon_{\mu\nu\alpha\beta} \varepsilon^\nu q^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi^a(u) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{S}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu} i\gamma_5(vx) q(0) | 0 \rangle &= -ie_q \langle \bar{q}q \rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_5 q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{A}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{V}(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q}q \rangle \left\{ \left[ \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left( g_{\alpha\nu} - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) \right) q_\beta \right. \right. \\
&\quad - \left( \varepsilon_\mu - q_\mu \frac{\varepsilon x}{qx} \right) \left( g_{\beta\nu} - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) \right) q_\alpha \\
&\quad - \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left( g_{\alpha\mu} - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) \right) q_\beta \\
&\quad + \left( \varepsilon_\nu - q_\nu \frac{\varepsilon x}{qx} \right) \left( g_{\beta\mu} - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) \right) q_\alpha \left. \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_1(\alpha_i) \\
&\quad + \left[ \left( \varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left( g_{\mu\beta} - \frac{1}{qx} (q_\mu x_\beta + q_\beta x_\mu) \right) q_\nu \right. \\
&\quad - \left( \varepsilon_\alpha - q_\alpha \frac{\varepsilon x}{qx} \right) \left( g_{\nu\beta} - \frac{1}{qx} (q_\nu x_\beta + q_\beta x_\nu) \right) q_\mu \\
&\quad - \left( \varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left( g_{\mu\alpha} - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) \right) q_\nu \\
&\quad + \left( \varepsilon_\beta - q_\beta \frac{\varepsilon x}{qx} \right) \left( g_{\nu\alpha} - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) \right) q_\mu \left. \right] \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_2(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_3(\alpha_i) \\
&\quad + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}} + v\alpha_g)qx} \mathcal{T}_4(\alpha_i) \left. \right\},
\end{aligned}$$

where  $\varphi_\gamma(u)$  is the leading twist 2,  $\psi^v(u)$ ,  $\psi^a(u)$ ,  $\mathcal{A}$  and  $\mathcal{V}$  are the twist 3 and  $h_\gamma(u)$ ,  $\mathbb{A}$ ,  $\mathcal{T}_i$  ( $i = 1, 2, 3, 4$ ) are the twist 4 photon DAs, respectively and  $\chi$  is the magnetic susceptibility of the quarks. The photon DAs is calculated in [10]. The measure  $\mathcal{D}\alpha_i$  is defined as

$$\int \mathcal{D}\alpha_i = \int_0^1 d\alpha_{\bar{q}} \int_0^1 d\alpha_q \int_0^1 d\alpha_g \delta(1 - \alpha_{\bar{q}} - \alpha_q - \alpha_g).$$

Explicit form of the photon DAs entering into above matrix elements.

$$\begin{aligned} \varphi_\gamma(u) &= 6u\bar{u} \left( 1 + \varphi_2(\mu) C_2^{\frac{3}{2}}(u - \bar{u}) \right), \\ \psi^v(u) &= 3 \left( 3(2u - 1)^2 - 1 \right) + \frac{3}{64} (15w_\gamma^V - 5w_\gamma^A) (3 - 30(2u - 1)^2 + 35(2u - 1)^4), \\ \psi^a(u) &= (1 - (2u - 1)^2) (5(2u - 1)^2 - 1) \frac{5}{2} \left( 1 + \frac{9}{16} w_\gamma^V - \frac{3}{16} w_\gamma^A \right), \\ \mathcal{A}(\alpha_i) &= 360\alpha_q\alpha_{\bar{q}}\alpha_g^2 \left( 1 + w_\gamma^A \frac{1}{2} (7\alpha_g - 3) \right), \\ \mathcal{V}(\alpha_i) &= 540w_\gamma^V (\alpha_q - \alpha_{\bar{q}}) \alpha_q \alpha_{\bar{q}} \alpha_g^2, \\ h_\gamma(u) &= -10 (1 + 2\kappa^+) C_2^{\frac{1}{2}}(u - \bar{u}), \\ \mathbb{A}(u) &= 40u^2\bar{u}^2 (3\kappa - \kappa^+ + 1) \\ &\quad + 8(\zeta_2^+ - 3\zeta_2) [u\bar{u}(2 + 13u\bar{u}) \\ &\quad + 2u^3(10 - 15u + 6u^2) \ln(u) + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln(\bar{u})], \\ \mathcal{T}_1(\alpha_i) &= -120(3\zeta_2 + \zeta_2^+) (\alpha_{\bar{q}} - \alpha_q) \alpha_{\bar{q}} \alpha_q \alpha_g, \\ \mathcal{T}_2(\alpha_i) &= 30\alpha_g^2 (\alpha_{\bar{q}} - \alpha_q) ((\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+) (1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\ \mathcal{T}_3(\alpha_i) &= -120(3\zeta_2 - \zeta_2^+) (\alpha_{\bar{q}} - \alpha_q) \alpha_{\bar{q}} \alpha_q \alpha_g, \\ \mathcal{T}_4(\alpha_i) &= 30\alpha_g^2 (\alpha_{\bar{q}} - \alpha_q) ((\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+) (1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)), \\ \mathcal{S}(\alpha_i) &= 30\alpha_g^2 \{ (\kappa + \kappa^+) (1 - \alpha_g) + (\zeta_1 + \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \\ &\quad + \zeta_2 [3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \}, \\ \tilde{\mathcal{S}}(\alpha_i) &= -30\alpha_g^2 \{ (\kappa - \kappa^+) (1 - \alpha_g) + (\zeta_1 - \zeta_1^+) (1 - \alpha_g) (1 - 2\alpha_g) \\ &\quad + \zeta_2 [3(\alpha_{\bar{q}} - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \}. \end{aligned}$$

The constants entering the above DAs are obtained as [10]  $\varphi_2(1 \text{ GeV}) = 0$ ,  $w_\gamma^V = 3.8 \pm 1.8$ ,  $w_\gamma^A = -2.1 \pm 1.0$ ,  $\kappa = 0.2$ ,  $\kappa^+ = 0$ ,  $\zeta_1 = 0.4$ ,  $\zeta_2 = 0.3$ ,  $\zeta_1^+ = 0$  and  $\zeta_2^+ = 0$ .

## Appendix B

In this Appendix we present the explicit expressions of the functions  $\Pi_i(u, d, s)$  for the magnetic moment of the  $\Sigma^{0*}$  baryon entering into the sum rule.

$$e^{m_{\Sigma^0}^2/M^2} \Pi_1(u, d, s) =$$

$$\begin{aligned}
& \frac{e}{48} \left[ (e_d - e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle (1 - t^2) - (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1 - t)^2 + (e_u - e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle (1 - t^2) \right] \\
& \times \left[ i_2(\mathcal{T}_3, 1) - 2i_2(\mathcal{T}_3, v) - i_2(\mathcal{T}_4, 1) + 2i_2(\mathcal{T}_4, v) \right] \\
& + \frac{e}{192} f_{3\gamma} \left\{ m_d \langle \bar{d}d \rangle \left[ e_s(1 + 6t + t^2) - e_u(1 + t)^2 \right] + m_s \langle \bar{s}s \rangle (e_u + e_d)(1 + 6t + t^2) \right. \\
& - m_u \langle \bar{u}u \rangle \left[ e_d(1 + t)^2 - e_s(1 + 6t + t^2) \right] \left. \right\} \left[ i_3(\mathcal{A}, 1) - 2i_3(\mathcal{A}, v) \right] \\
& - \frac{e}{48} \left[ 3(e_d + e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle (1 - t^2) + (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1 - t)^2 + 3(e_u + e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle (1 - t^2) \right] i_2(\mathcal{S}, 1) \\
& - \frac{e}{192} f_{3\gamma} \left\{ m_d \langle \bar{d}d \rangle \left[ e_s(3 + 2t + 3t^2) - e_u(1 + t)^2 \right] + m_s \langle \bar{s}s \rangle (e_u + e_d)(3 + 2t + 3t^2) \right. \\
& - m_u \langle \bar{u}u \rangle \left[ e_d(1 + t)^2 - e_s(3 + 2t + 3t^2) \right] \left. \right\} i_3(\mathcal{V}, 1) \\
& + \frac{e}{96} f_{3\gamma} \left\{ \langle \bar{d}d \rangle \left[ -e_s m_d(1 + t)^2 + e_u m_d(3 + 2t + 3t^2) + 6e_u m_s(1 - t^2) + 2e_s m_u(1 - t)^2 \right] \right. \\
& + \langle \bar{s}s \rangle \left[ 6(e_u m_d + e_d m_u)(1 - t^2) + m_s(e_d + e_u)(3 + 2t + 3t^2) \right] \\
& + \langle \bar{u}u \rangle \left[ 2e_s m_d(1 - t)^2 + 6e_d m_s(1 - t^2) + e_d m_u(3 + 2t + 3t^2) - e_s m_u(1 + t)^2 \right] \left. \right\} \psi^a(u_0) \\
& - \frac{e}{96} (1 - t) \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle (e_u - e_s)(1 + t) + \langle \bar{d}d \rangle \left[ (e_d - e_s) \langle \bar{s}s \rangle (1 + t) - (e_u + e_d) \langle \bar{u}u \rangle (1 - t) \right] \right\} \mathbb{A}(u_0) \\
& - \frac{e}{1152\pi^2} (1 - t) \left\{ e_s(m_u + m_d) \langle \bar{s}s \rangle \langle g^2 G^2 \rangle - \langle \bar{u}u \rangle \left[ e_u \langle g^2 G^2 \rangle \left( -m_d(1 - t) + m_s(1 + t) \right) \right. \right. \\
& - 2\pi^2(e_u - 7e_s) m_0^2 \langle \bar{s}s \rangle (1 + t) \left. \right] + \langle \bar{d}d \rangle \left[ -e_d \langle g^2 G^2 \rangle \left( m_s(1 + t) - m_u(1 - t) \right) \right. \\
& + 2\pi^2(e_d - 7e_s) m_0^2 \langle \bar{s}s \rangle (1 + t) - 6\pi^2(e_u + e_d) m_0^2 \langle \bar{u}u \rangle (1 - t) \left. \right] \left. \right\} \chi\varphi_\gamma(u_0) \\
& - \frac{e}{768\pi^2} \left\{ m_0^2 \langle \bar{s}s \rangle \left[ -18(e_u m_d + e_d m_u)(1 - t^2) + m_s(e_u + e_d)(5 + 2t + 5t^2) \right] \right. \\
& + \langle \bar{u}u \rangle \left[ m_0^2 \left( -6e_s m_d(1 - t)^2 - 18e_d m_s(1 - t^2) + (e_d + e_s) m_u(5 + 2t + 5t^2) \right) - 96\pi^2 e_d \langle \bar{s}s \rangle (1 - t^2) \right] \\
& + \langle \bar{d}d \rangle \left[ e_u m_0^2 \left( m_d(5 + 2t + 5t^2) - 18m_s(1 - t^2) \right) + e_s m_0^2 \left( m_d(5 + 2t + 5t^2) - 6m_u(1 - t)^2 \right) \right] \\
& - 96\pi^2 e_u \langle \bar{s}s \rangle (1 - t^2) - 32\pi^2 e_s \langle \bar{u}u \rangle (1 - t)^2 \left. \right\} \\
& + \frac{e}{768\pi^2} \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left\{ (1 - t)^2 \left[ 3m_0^2(e_u m_u \langle \bar{d}d \rangle + e_d m_d \langle \bar{u}u \rangle) + \langle g^2 G^2 \rangle (e_u m_d \langle \bar{u}u \rangle + e_d m_u \langle \bar{d}d \rangle) \chi\varphi_\gamma(u_0) \right] \right. \\
& + (1 - t^2) m_0^2 \left[ (18e_u + 7e_s) m_s \langle \bar{d}d \rangle + (18e_d + 7e_s) m_s \langle \bar{u}u \rangle + e_d(18m_u - m_d) \langle \bar{s}s \rangle - e_d(m_u - 18m_d) \langle \bar{s}s \rangle \right] \\
& + (1 - t^2) \langle g^2 G^2 \rangle \left[ e_s(m_u + m_d) \langle \bar{s}s \rangle - m_s(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \right] \chi\varphi_\gamma(u_0) \left. \right\} \\
& - \frac{e}{64\pi^2} M^2 \langle \bar{d}d \rangle \left\{ -e_s \left[ m_d(1 + t)^2 + 2m_s(1 - t^2) \right] + 2e_s m_u(1 - t)^2 \right. \\
& + e_u \left[ m_d(3 + 2t + 3t^2) + 6m_s(1 - t^2) + 2m_u(1 - t)^2 \right] \left. \right\} \\
& - \frac{e}{64\pi^2} M^2 \langle \bar{s}s \rangle \left\{ -e_d \left[ 2m_d(1 - t^2) - m_s(3 + 2t + 3t^2) - 6m_u(1 - t^2) \right] \right. \\
& + e_u \left[ 6m_d(1 - t^2) + m_s(3 + 2t + 3t^2) - 2m_u(1 - t^2) \right] \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{e}{64\pi^2} M^2 \langle \bar{u}u \rangle \left\{ 2(e_d + e_s)m_d(1-t)^2 + 2(3e_d + e_s)m_s(1-t^2) + m_u \left[ e_d(3+2t+3t^2) - e_s(1+t)^2 \right] \right\} \\
& + \frac{e}{128\pi^2} M^2 (1-t) \left\{ e_d \langle \bar{d}d \rangle \left[ -m_u(1-t) + m_s(1+t) \right] - e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& - e_u \langle \bar{u}u \rangle \left[ m_d(1-t) - m_s(1+t) \right] \left. \right\} \mathbb{A}(u_0) \\
& + \frac{e}{32\pi^2} M^2 (1-t) \left\{ e_d \langle \bar{d}d \rangle \left[ m_u(1-t) + 2m_s(1+t) \right] + e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& + e_u \langle \bar{u}u \rangle \left[ m_d(1-t) + 2m_s(1+t) \right] \left. \right\} i_2(\mathcal{S}, 1) \\
& - \frac{e}{64\pi^2} M^2 (1-t) \left\{ e_d \langle \bar{d}d \rangle \left[ -m_u(1-t) + m_s(1+t) \right] - e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& + e_u \langle \bar{u}u \rangle \left[ -m_d(1-t) + m_s(1+t) \right] \left. \right\} \left[ i_2(\tilde{\mathcal{S}}, 1) + i_2(\mathcal{T}_3, 1) - 2i_2(\mathcal{T}_3, v) - i_2(\mathcal{T}_4, 1) + 2i_2(\mathcal{T}_4, v) \right] \\
& + \frac{e}{24} M^2 (1-t) \left\{ (e_d - e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle (1+t) - \langle \bar{u}u \rangle \left[ (e_u + e_d) \langle \bar{d}d \rangle (1-t) - (e_u - e_s) \langle \bar{s}s \rangle (1+t) \right] \right\} \chi \varphi_\gamma(u_0) \\
& + \frac{e}{64\pi^2} (1-t) M^2 \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left\{ -2 \langle \bar{d}d \rangle \left[ e_u m_u(1-t) + e_s m_s(1+t) \right] \right. \\
& + 2(e_u m_u + e_d m_d) \langle \bar{s}s \rangle (1+t) - 2 \langle \bar{u}u \rangle \left[ e_d m_d(1-t) + e_s m_s(1+t) \right] \\
& + e_d \langle \bar{d}d \rangle \left[ m_u(1-t) + m_s(1+t) \right] - e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) + e_u \langle \bar{u}u \rangle \left[ m_d(1-t) + m_s(1+t) \right] i_2(\mathcal{S}, 1) \\
& + e_d \langle \bar{d}d \rangle \left[ m_u(1-t) - m_s(1+t) \right] + e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) + e_u \langle \bar{u}u \rangle \left[ m_d(1-t) - m_s(1+t) \right] i_2(\tilde{\mathcal{S}}, 1) \left. \right\} \\
& - \frac{e}{1536\pi^2} \frac{1}{M^2} (1-t) \langle g^2 G^2 \rangle \left\{ e_d \langle \bar{d}d \rangle \left[ m_u(1-t) + 3m_s(1+t) \right] + 3e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& + e_u \langle \bar{u}u \rangle \left[ m_d(1-t) + 3m_s(1+t) \right] \left. \right\} i_2(\mathcal{S}, 1) \\
& + \frac{e}{1536\pi^2} \frac{1}{M^2} (1-t) \langle g^2 G^2 \rangle \left\{ e_d \langle \bar{d}d \rangle \left[ -m_u(1-t) + m_s(1+t) \right] - e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& - e_u \langle \bar{u}u \rangle \left[ m_d(1-t) - m_s(1+t) \right] \left. \right\} \left[ i_2(\mathcal{T}_3, 1) - 2i_2(\mathcal{T}_3, v) - i_2(\mathcal{T}_4, 1) + 2i_2(\mathcal{T}_4, v) \right] \\
& + \frac{e}{64} \frac{1}{M^2} f_{3\gamma} m_0^2 (1-t^2) \left[ m_s(e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) + (e_u m_d + e_d m_u) \langle \bar{s}s \rangle \right] \psi^a(u_0) \\
& + \frac{e}{3072\pi^2} \frac{1}{M^2} \langle g^2 G^2 \rangle (1-t) \left\{ e_d \langle \bar{d}d \rangle \left[ m_u(1-t) - m_s(1+t) \right] + e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) \right. \\
& + e_u \langle \bar{u}u \rangle \left[ m_d(1-t) - m_s(1+t) \right] \left. \right\} \mathbb{A}(u_0) \\
& + \frac{e}{768\pi^2} \frac{1}{M^2} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) + e_s m_u(1-t) \right] + 3 \langle \bar{s}s \rangle (e_u m_d + e_d m_u) (1+t) \right. \\
& + \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) + e_s m_d(1-t) \right] \left. \right\} \\
& + \frac{e}{9216\pi^2} \frac{1}{M^4} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) + e_s m_u(1-t) \right] + 3(e_u m_d + e_d m_u) \langle \bar{s}s \rangle (1+t) \right. \\
& + \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) + e_s m_d(1-t) \right] \left. \right\} \left[ 3m_0^2 + 8\pi^2 f_{3\gamma} \psi^a(u_0) \right] \\
& + \frac{e}{2304} \frac{1}{M^6} f_{3\gamma} m_0^2 \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) + e_s m_u(1-t) \right] + 3(e_u m_d + e_d m_u) \langle \bar{s}s \rangle (1+t) \right.
\end{aligned}$$

$$\begin{aligned}
& + \langle \bar{u}u \rangle \left[ 3e_d m_s (1+t) + e_s m_d (1-t) \right] \Big\} \psi^a(u_0) \\
& + \frac{e}{128\pi^2} M^4 \left\{ f_{3\gamma} \left[ 2t(e_u + e_d + 3e_s) + e_s(1+t^2) \right] \left[ i_3(\mathcal{A}, 1) - 2i_3(\mathcal{A}, v) \right] \right. \\
& - f_{3\gamma} \left[ (e_u + e_d)(1+t^2) + e_s(3+2t+3t^2) \right] i_3(\mathcal{V}, 1) - 2(1-t) \left[ e_d \langle \bar{d}d \rangle \left( -m_u(1-t) + m_s(1+t) \right) \right. \\
& - e_s(m_u + m_d) \langle \bar{s}s \rangle (1+t) + e_u \langle \bar{u}u \rangle \left( -m_d(1-t) + m_s(1+t) \right) \Big] \chi \varphi_\gamma(u_0) \\
& + f_{3\gamma} \left[ (e_u + e_d)(3+2t+3t^2) - e_s(1+t)^2 \right] \psi^a(u_0) \Big\} \\
& - \frac{e}{256\pi^4} M^6 \left[ (e_u + e_d)(3+2t+3t^2) - e_s(1+t)^2 \right]
\end{aligned}$$

$$\begin{aligned}
& e^{m_{\Sigma_0}^2/M^2} \Pi_2(u, d, s) = \\
& \frac{e}{192} \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 4(e_u + e_s) m_d (3+2t+3t^2) - e_u m_s (1-t^2) + e_s m_u (1-t)^2 \right] \right. \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 4(e_d + e_s) m_u (3+2t+3t^2) - e_d m_s (1-t^2) + e_s m_d (1-t)^2 \right] \\
& + \langle \bar{u}u \rangle \langle \bar{d}d \rangle \left[ (e_u m_d + e_d m_u) (1-t^2) - 4(e_u + e_d) m_s (1+t)^2 \right] \Big\} i_2(\mathcal{S}, 1) \\
& - \frac{e}{192} \left\{ \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 4m_u (e_d + e_s) (1+6t+t^2) + e_s m_d (1-t)^2 + e_d m_s (1-t^2) \right] \right. \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle \left[ (e_u m_d + e_d m_u) (1-t^2) + 4(e_u + e_d) m_s (1+t)^2 \right] \\
& + \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 4m_d (e_u + e_s) (1+6t+t^2) + e_s m_u (1-t)^2 + e_u m_s (1-t^2) \right] \Big\} i_2(\tilde{\mathcal{S}}, 1) \\
& + \frac{e}{192} \left\{ \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u) (1-t^2) - \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s (1-t^2) + e_s m_u (1-t)^2 \right] \right. \\
& - \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_s m_d (1-t)^2 + e_d m_s (1-t^2) \right] \Big\} \left[ i_2(\mathcal{T}_2, 1) - 2i_2(\mathcal{T}_2, v) \right] \\
& + \frac{e}{96} e_s (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle (1-t)^2 \left[ i_2(\mathcal{T}_3, 1) - 2i_2(\mathcal{T}_3, v) \right] \\
& + \frac{e}{192} \left\{ -\langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u) (1-t^2) + \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s (1-t^2) - e_s m_u (1-t)^2 \right] \right. \\
& - \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_s m_d (1-t)^2 - e_d m_s (1-t^2) \right] \Big\} \left[ i_2(\mathcal{T}_4, 1) - 2i_2(\mathcal{T}_4, v) \right] \\
& - \frac{e}{3072\pi^2} f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \left\{ e_u \left[ m_d (1+t) - m_s (1-t) \right] + e_d \left[ m_u (1+t) - m_s (1-t) \right] \right. \\
& - e_s (m_u + m_d) (1+t) \Big\} \left[ i_3(\mathcal{A}, 1) - 2i_3(\mathcal{A}, v) \right] \\
& - \frac{e}{384} (1-t) \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s (1+t) + e_s m_u (1-t) \right] + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d m_s (1+t) + e_s m_d (1-t) \right] \right. \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u) (1+t) \Big\} \left[ i_3(\tilde{\mathcal{S}}, 1) + i_3(\mathcal{T}_2, 1) - 2i_3(\mathcal{T}_2, v) \right] \\
& + \frac{e}{192} e_s (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle (1-t)^2 \left[ i_3(\mathcal{T}_3, 1) - 2i_3(\mathcal{T}_3, v) \right] \\
& - \frac{e}{384} (1-t) \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s (1+t) - e_s m_u (1-t) \right] + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d m_s (1+t) - e_s m_d (1-t) \right] \right.
\end{aligned}$$

$$\begin{aligned}
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u)(1+t) \Big\} \Big[ i_3(\mathcal{S}, 1) - i_3(\mathcal{T}_4, 1) + 2i_3(\mathcal{T}_4, v) \Big] \\
& + \frac{e}{3072\pi^2} f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \Big\{ e_u \Big[ m_d(1+t) + m_s(1-t) \Big] + e_d \Big[ m_u(1+t) + m_s(1-t) \Big] \\
& - e_s(m_u + m_d)(1+t) \Big\} i_3(\mathcal{V}, 1) \\
& + \frac{e}{96} \Big\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \Big[ 2e_u m_d(6+4t+6t^2) + 3e_u m_s(1-t^2) - e_s m_u(1-t)^2 - 2e_s m_d(1+t)^2 \Big] \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \Big[ e_d m_u(6+4t+6t^2) + 3e_d m_s(1-t^2) - e_s m_d(1-t)^2 - 2e_s m_u(1+t)^2 \Big] \\
& + \langle \bar{u}u \rangle \langle \bar{d}d \rangle \Big[ (e_u + e_d)m_s(6+4t+6t^2) + 3(e_u m_d + e_d m_u)(1-t^2) \Big] \Big\} \Big[ \tilde{j}_1(h_\gamma) - 2\tilde{j}_2(h_\gamma) \Big] \\
& - \frac{e}{576} m_0^2 \Big\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \Big[ 4e_u m_d(5+4t+5t^2) + e_s m_u(1-t)^2 + 2e_s m_d(1+t)^2 \Big] \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \Big[ 4e_d m_u(5+4t+5t^2) + e_s m_d(1-t)^2 + 2e_s m_u(1+t)^2 \Big] \\
& + \langle \bar{u}u \rangle \langle \bar{d}d \rangle \Big[ 4m_s(e_u + e_d)m_u(5+4t+5t^2) \Big] \Big\} \chi\varphi_\gamma(u_0) \\
& + \frac{e}{2304\pi^2} f_{3\gamma}(1-t) \Big\{ m_0^2 \pi^2(1+t) \Big[ (e_d - 7e_s) \langle \bar{u}u \rangle + (e_u - 7e_s) \langle \bar{d}d \rangle \Big] - 3(e_u + e_d) m_0^2 \pi^2 \langle \bar{s}s \rangle (1+t) \\
& + e_u \langle g^2 G^2 \rangle \Big[ m_d(1+t) - m_s(1-t) \Big] + e_d \langle g^2 G^2 \rangle \Big[ m_u(1+t) - m_s(1-t) \Big] \\
& - e_s \langle g^2 G^2 \rangle (1+t)(m_u + m_d) \Big\} \psi^a(u_0) \\
& - \frac{e}{9216\pi^2} f_{3\gamma}(1-t) \Big\{ 2m_0^2 \pi^2(1+t) \Big[ (e_d - 7e_s) \langle \bar{u}u \rangle + (e_u - 7e_s) \langle \bar{d}d \rangle \Big] - 6(e_u + e_d) m_0^2 \pi^2 \langle \bar{s}s \rangle (1+t) \\
& - e_u \langle g^2 G^2 \rangle \Big[ m_d(1+t) - m_s(1-t) \Big] - e_d \langle g^2 G^2 \rangle \Big[ m_u(1+t) - m_s(1-t) \Big] \\
& + e_s \langle g^2 G^2 \rangle (1+t)(m_u + m_d) \Big\} \psi^{a'}(u_0) \\
& - \frac{e}{384} \Big\{ 4e_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle \Big[ 4m_d(3+2t+3t^2) - m_s(1-t^2) + m_u(1-t)^2 \Big] \\
& + e_s m_u \langle \bar{u}u \rangle \langle \bar{s}s \rangle (1-t)^2 \Big[ 2\mathbb{A}(u_0) - \mathbb{A}'(u_0) \Big] \Big\} \\
& - \frac{e}{384} \Big\{ 4e_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle \Big[ 4m_u(3+2t+3t^2) - m_s(1-t^2) + m_d(1-t)^2 \Big] \\
& + e_s m_d \langle \bar{d}d \rangle \langle \bar{s}s \rangle (1-t)^2 \Big[ 2\mathbb{A}(u_0) - \mathbb{A}'(u_0) \Big] \Big\} \\
& - \frac{e}{96} e_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1+t) \Big\{ \Big[ (1-t)(m_u + m_d) - 4m_s(1+t) \Big] \Big\} \\
& + \frac{e}{1152} e_s m_0^2 \langle \bar{s}s \rangle (1-t)^2 (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \chi\varphi'_\gamma(u_0) \\
& + \frac{e}{512\pi^2} M^4 e_s \langle \bar{s}s \rangle (1-t)^2 \Big\{ 2\mathbb{A}(u_0) + \mathbb{A}'(u_0) - 2 \Big[ 6i_2(\mathcal{S}, 1) + 2i_2(\tilde{\mathcal{S}}, 1) + 2i_2(\mathcal{T}_2, 1) \\
& + 4i_2(\mathcal{T}_3, 1) - 6i_2(\mathcal{T}_4, 1) - 4i_2(\mathcal{T}_2, v) - 8i_2(\mathcal{T}_3, v) + 12i_2(\mathcal{T}_4, v) - i_3(\mathcal{S}, 1) + i_3(\tilde{\mathcal{S}}, 1) \\
& + i_3(\mathcal{T}_2, 1) - 2i_3(\mathcal{T}_3, 1) + i_3(\mathcal{T}_4, 1) - 2i_3(\mathcal{T}_2, v) + 4i_3(\mathcal{T}_3, v) - 2i_3(\mathcal{T}_4, v) + 2\tilde{j}_1(h_\gamma) - 4\tilde{j}_2(h_\gamma) \Big] \Big\} \\
& + \frac{e}{512\pi^2} M^4 f_{3\gamma}(1-t) \Big\{ e_u \Big[ m_d(1+t) - m_s(1-t) \Big] + e_d \Big[ m_u(1+t) - m_s(1-t) \Big] \\
& - e_s(m_u + m_d)(1+t) \Big\} \Big[ 4i_3(\mathcal{A}, 1) - 8i_3(\mathcal{A}, v) - 6\psi^a(u_0) - \psi^{a'}(u_0) \Big]
\end{aligned}$$

$$\begin{aligned}
& + \frac{3e}{128\pi^2} M^4 (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) (1-t^2) \left[ \tilde{j}_1(h_\gamma) - 2\tilde{j}_2(h_\gamma) \right] \\
& + \frac{3e}{128\pi^2} M^4 (1-t) \left\{ (1+t) \left[ (e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) \mathbb{A}(u_0) - (e_d - e_s) \langle \bar{u}u \rangle - (e_u - e_s) \langle \bar{d}d \rangle \right] \right. \\
& + (e_u + e_d) \langle \bar{s}s \rangle (1-t) \left. \right\} \\
& - \frac{e}{128\pi^2} M^4 f_{3\gamma} (1-t) \left\{ e_u \left[ 5m_d(1+t) + m_s(1-t) \right] + e_d \left[ 5m_u(1+t) + m_s(1-t) \right] \right. \\
& + e_s(m_u + m_d)(1+t) \left. \right\} i_3(\mathcal{V}, 1) \\
& + \frac{e}{64\pi^2} M^4 f_{3\gamma} (1-t^2) \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left[ 2(e_u m_d + e_d m_u) + e_s(m_u + m_d) \right] i_3(\mathcal{V}, 1) \\
& + \frac{e}{6144\pi^2} f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left\{ e_u \left[ m_d(1+t) - m_s(1-t) \right] \right. \\
& + e_d \left[ m_u(1+t) - m_s(1-t) \right] - e_s(m_u + m_d)(1+t) \left. \right\} \left[ 2\psi^a(u_0) - \psi^{a'}(u_0) \right] \\
& - \frac{e}{2048\pi^4} M^2 \langle g^2 G^2 \rangle (1-t) \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left\{ e_u \left[ m_d(1+t) - m_s(1-t) \right] + e_d \left[ m_u(1+t) - m_s(1-t) \right] \right. \\
& - e_s(m_u + m_d)(1+t) \left. \right\} \\
& - \frac{e}{192} M^2 f_{3\gamma} (1-t) \left\{ e_u \left[ \langle \bar{d}d \rangle (1+t) - \langle \bar{s}s \rangle (1-t) \right] + e_d \left[ \langle \bar{u}u \rangle (1+t) - \langle \bar{s}s \rangle (1-t) \right] \right. \\
& - e_s(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (1+t) \left. \right\} \left\{ 2i_3(\mathcal{A}, 1) - 2 \left[ i_3(\mathcal{V}, 1) + 2i_3(\mathcal{A}, v) + \psi^a(u_0) \right] - \psi^{a'}(u_0) \right\} \\
& + \frac{e}{6144\pi^4} M^2 (1-t) \left\{ 4e_u m_0^2 \pi^2 \left[ \langle \bar{d}d \rangle (1+t) - 3\langle \bar{s}s \rangle (1-t) \right] - 5e_u \langle g^2 G^2 \rangle \left[ m_d(1+t) - m_s(1-t) \right] \right. \\
& + 4e_d m_0^2 \pi^2 \left[ \langle \bar{u}u \rangle (1+t) - 3\langle \bar{s}s \rangle (1-t) \right] - 5e_d \langle g^2 G^2 \rangle \left[ m_u(1+t) - m_s(1-t) \right] \\
& - e_s(1+t) \left[ 28m_0^2 \pi^2 (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) - 5\langle g^2 G^2 \rangle (m_u + m_d) \right] \left. \right\} \\
& + \frac{e}{48} M^2 (1-t)^2 (e_u + e_d) f_{3\gamma} \langle \bar{s}s \rangle i_3(\mathcal{V}, 1) \\
& + \frac{e}{48} M^2 \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 2e_s m_d(1+t)^2 - e_u m_d(6+4t+6t^2) - 3e_u m_s(1-t^2) \right] \right. \\
& - \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 3e_d m_s(1-t^2) + e_d m_u(6+4t+6t^2) - 2e_s m_u(1+t)^2 \right] \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle \left[ 3e_u m_d(1-t^2) + (e_u + e_d) m_s(6+4t+6t^2) + 3e_d m_u(1-t^2) \right] \left. \right\} \chi \varphi_\gamma(u_0) \\
& - \frac{e}{96} M^2 e_s (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle (1-t)^2 \chi \varphi'_\gamma(u_0) \\
& - \frac{e}{256\pi^4} M^6 \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) (1-t) \left[ 9(e_u m_u + e_d m_d)(1+t) + 2e_s m_s(1-t) \right] \\
& - \frac{e}{1536\pi^4} M^6 (1-t) \left\{ 3e_u \left[ (1+t)(27m_u - 5m_d) + 5m_s(1-t) \right] \right. \\
& + 3e_d \left[ (1+t)(27m_d - 5m_u) + 5m_s(1-t) \right] + e_s \left[ 15(1+t)(m_u + m_d) + 16m_s(1-t) \right] \left. \right\} \\
& - \frac{e}{192\pi^2} M^6 (1-t) \left[ 9(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) (1+t) + 2e_s \langle \bar{s}s \rangle (1-t) \right] \chi \varphi_\gamma(u_0)
\end{aligned}$$



$$\begin{aligned}
& - \frac{e}{384\pi^2} M^6 (1-t)^2 e_s \langle \bar{s}s \rangle \chi \varphi'_\gamma(u_0) \\
& - \frac{e}{1152} \frac{1}{M^2} m_0^2 \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 2e_u m_d (5+4t+5t^2) + e_s m_d (1+t)^2 \right] \right. \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 2e_d m_u (5+4t+5t^2) + e_s m_u (1+t)^2 \right] \\
& + 2(e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (5+4t+5t^2) \left. \right\} \mathbb{A}(u_0) \\
& + \frac{e}{192} \frac{1}{M^2} m_0^2 \left\{ (3+2t+3t^2) \left[ \langle \bar{u}u \rangle \langle \bar{s}s \rangle (e_u + e_s) m_d + \langle \bar{d}d \rangle \langle \bar{s}s \rangle (e_d + e_s) m_u \right] \right. \\
& - (e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1+t)^2 \left. \right\} i_2(\mathcal{S}, 1) \\
& - \frac{e}{192} \frac{1}{M^2} m_0^2 \left\{ (1+6t+t^2) \left[ \langle \bar{u}u \rangle \langle \bar{s}s \rangle (e_u + e_s) m_d + \langle \bar{d}d \rangle \langle \bar{s}s \rangle (e_d + e_s) m_u \right] \right. \\
& - (e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1+t)^2 \left. \right\} i_2(\tilde{\mathcal{S}}, 1) \\
& - \frac{e}{576} \frac{1}{M^2} \langle g^2 G^2 \rangle \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_s m_d (1+t)^2 - e_u m_d (3+2t+3t^2) \right] \right. \\
& - \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_s m_u (1+t)^2 - e_d m_u (3+2t+3t^2) \right] \\
& + \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u + e_d) m_s (3+2t+3t^2) \left. \right\} \chi \varphi_\gamma(u_0) \\
& - \frac{e}{576} \frac{1}{M^2} m_0^2 \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 2e_u m_d (5+4t+5t^2) + e_s m_d (1+t)^2 \right] \right. \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 2e_d m_u (5+4t+5t^2) + e_s m_u (1+t)^2 \right] \\
& + 2(e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (5+4t+5t^2) \left. \right\} \left[ \tilde{j}_1(h_\gamma) - 2\tilde{j}_2(h_\gamma) \right] \\
& - \frac{e}{48} \frac{1}{M^2} m_0^2 \left[ (e_u m_u \langle \bar{d}d \rangle + e_d m_d \langle \bar{u}u \rangle) \langle \bar{s}s \rangle (3+2t+3t^2) - e_s m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1+t)^2 \right] \\
& + \frac{e}{1152} \frac{1}{M^4} \langle g^2 G^2 \rangle \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_s m_d (1+t)^2 - e_u m_d (3+2t+3t^2) \right] \right. \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_s m_u (1+t)^2 - e_d m_u (3+2t+3t^2) \right] \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u + e_d) m_s (3+2t+3t^2) \left. \right\} \left[ \mathbb{A}(u_0) + m_0^2 \chi \varphi_\gamma(u_0) - \tilde{j}_1(h_\gamma) + 2\tilde{j}_2(h_\gamma) \right] \\
& + \frac{e}{4608} \frac{1}{M^6} \langle g^2 G^2 \rangle m_0^2 \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_s m_d (1+t)^2 - e_u m_d (3+2t+3t^2) \right] \right. \\
& + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_s m_u (1+t)^2 - e_d m_u (3+2t+3t^2) \right] \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u + e_d) m_s (3+2t+3t^2) \left. \right\} \left[ 3\mathbb{A}(u_0) - 2\tilde{j}_1(h_\gamma) + 4\tilde{j}_2(h_\gamma) \right]
\end{aligned}$$

$$\begin{aligned}
& e^{m_{\Sigma_0}^2/M^2} \Pi_3(u, d, s) = \\
& - \frac{e}{6144\pi^2} \langle g^2 G^2 \rangle (1-t) \left\{ e_u \langle \bar{u}u \rangle \left[ m_d (1-t) - m_s (1+t) \right] + e_d \langle \bar{d}d \rangle \left[ m_u (1-t) - m_s (1+t) \right] \right. \\
& + e_s (m_u + m_d) \langle \bar{s}s \rangle (1+t) \left. \right\} \left\{ 4i_3(\mathcal{S}, 1) + 2 \left[ i_3(\mathcal{T}_1, 1) + i_3(\mathcal{T}_3, 1) \right] \right.
\end{aligned}$$

$$\begin{aligned}
& - 2i_3(\mathcal{T}_4, 1) - 2i_3(\mathcal{S}, v) - 2i_3(\mathcal{T}_3, v) + 2i_3(\mathcal{T}_4, v) + \mathbb{A}'(u_0) \Big\} \\
& + \frac{e}{128} f_{3\gamma} m_0^2 (1-t^2) \left[ e_u(m_d \langle \bar{s}s \rangle + m_s \langle \bar{d}d \rangle) + e_d(m_u \langle \bar{s}s \rangle + m_s \langle \bar{u}u \rangle) \right] \left[ 4\psi^v(u_0) - \psi^{a'}(u_0) \right] \\
& - \frac{e}{1536\pi^2} \langle g^2 G^2 \rangle (1-t^2) \left\{ \left[ m_s(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) - e_s(m_u + m_d) \langle \bar{s}s \rangle \right] \left[ i_3(\mathcal{S}, 1) + i_3(\mathcal{T}_1, 1) \right. \right. \\
& - i_3(\mathcal{T}_4, 1) \Big] + 2 \left[ m_s(e_u \langle \bar{u}u \rangle + e_d \langle \bar{d}d \rangle) + 2e_s(m_u + m_d) \langle \bar{s}s \rangle \right] i_3(\mathcal{S}, v) \Big\} \\
& - \frac{e}{2304\pi^2} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{u}u \rangle \left[ 3e_d m_s (1+t) - e_s m_d (1-t) \right] + \langle \bar{d}d \rangle \left[ 3e_u m_s (1+t) - e_s m_u (1-t) \right] \right. \\
& + 3(e_u m_d + e_d m_u) \langle \bar{s}s \rangle (1+t) \Big\} \\
& - \frac{e}{48} m_0^2 (1-t) \left[ 3(1+t)(e_u \langle \bar{d}d \rangle + e_d \langle \bar{u}u \rangle) \langle \bar{s}s \rangle - (1-t)e_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle \right] \\
& - \frac{e}{256\pi^2} M^4 (1-t) \left\{ e_u \langle \bar{u}u \rangle \left[ m_d (1-t) - m_s (1+t) \right] + e_d \langle \bar{d}d \rangle \left[ m_u (1-t) - m_s (1+t) \right] \right. \\
& + e_s \langle \bar{s}s \rangle (m_u + m_d) \langle \bar{s}s \rangle (1+t) \Big\} \left\{ 2i_3(\mathcal{S}, 1) - 2 \left[ i_3(\tilde{\mathcal{S}}, 1) - 2i_3(\mathcal{T}_1, 1) + i_3(\mathcal{T}_2, 1) + i_3(\mathcal{T}_4, 1) \right. \right. \\
& - 6i_3(\mathcal{S}, v) - 2i_3(\mathcal{T}_3, v) + 2i_3(\mathcal{T}_4, v) \Big] - \mathbb{A}'(u_0) \Big\} \\
& + \frac{e}{32\pi^2} M^4 (1-t)^2 (e_u m_d \langle \bar{u}u \rangle + e_d m_u \langle \bar{d}d \rangle) \left[ i_3(\mathcal{S}, 1) + i_3(\mathcal{T}_1, 1) - i_3(\mathcal{T}_4, 1) + i_3(\mathcal{S}, v) \right] \\
& + \frac{3e}{32\pi^2} M^4 (1-t^2) e_s (m_u + m_d) \langle \bar{s}s \rangle i_3(\mathcal{S}, v) \\
& - \frac{e}{48} M^4 (1-t) \left\{ \left[ (e_u - e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle + (e_d - e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle \right] (1+t) - (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1-t) \right\} \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{128\pi^2} M^4 \langle \bar{u}u \rangle \left\{ 2(e_d - e_s)(1-t) \left[ m_d (1-t) + 3m_s (1+t) \right] + m_u \left[ e_d (1+t)^2 - e_s (7+6t+7t^2) \right] \right\} \\
& + \frac{e}{128\pi^2} M^4 \langle \bar{d}d \rangle \left\{ e_u \left[ -2m_u (1-t) + m_d (1+t) + 6m_s (1-t) \right] (1+t) - e_s m_d (7+6t+7t^2) \right. \\
& - 2e_s \left[ m_u (1-t) + 3m_s (1+t) \right] (1-t) \Big\} \\
& - \frac{e}{128\pi^2} M^4 \langle \bar{s}s \rangle (1+t) \left\{ e_u \left[ 6(m_u - m_d)(1-t) - m_s (1+t) \right] - e_d \left[ 6(m_u - m_d)(1-t) + m_s (1+t) \right] \right\} \\
& + \frac{e}{128\pi^2} M^4 \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) (1-t) \left\{ e_u \langle \bar{u}u \rangle \left[ m_d (1-t) + m_s (1+t) \right] \right. \\
& + e_d \langle \bar{d}d \rangle \left[ m_u (1-t) + m_s (1+t) \right] + e_s \langle \bar{s}s \rangle (1+t) (m_u + m_d) \Big\} \\
& \times \left[ i_3(\mathcal{S}, 1) - i_3(\tilde{\mathcal{S}}, 1) + i_3(\mathcal{T}_1, 1) - i_3(\mathcal{T}_2, 1) + i_3(\mathcal{T}_3, 1) - i_3(\mathcal{T}_4, 1) \right] \\
& + \frac{e}{64\pi^2} M^4 \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) (1-t)^2 (e_u m_d \langle \bar{u}u \rangle + e_d m_u \langle \bar{d}d \rangle) \left[ i_3(\tilde{\mathcal{S}}, 1) + i_3(\mathcal{T}_2, 1) - i_3(\mathcal{T}_3, 1) \right] \\
& - \frac{e}{768\pi^2} \langle g^2 G^2 \rangle \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) (1-t) \left\{ 3(1+t) \left[ e_u(m_d \langle \bar{s}s \rangle + m_s \langle \bar{d}d \rangle) + e_d(m_u \langle \bar{s}s \rangle + m_s \langle \bar{u}u \rangle) \right] \right. \\
& - (1-t)e_s(m_u \langle \bar{d}d \rangle + m_d \langle \bar{u}u \rangle) \Big\} \\
& - \frac{e}{1536\pi^2} M^2 \langle g^2 G^2 \rangle \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) (1-t) \left\{ e_u \langle \bar{u}u \rangle \left[ m_d (1-t) - m_s (1+t) \right] \right.
\end{aligned}$$

$$\begin{aligned}
& + e_d \langle \bar{d}d \rangle \left[ m_u(1-t) - m_s(1+t) \right] + e_s \langle \bar{s}s \rangle (m_u + m_d)(1+t) \Big\} \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{96} M^2 (1-t) \left\{ (1+t) \left[ (e_u - e_s) \langle \bar{u}u \rangle + (e_d - e_s) \langle \bar{d}d \rangle \right] \langle \bar{s}s \rangle \right. \\
& - (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1-t) \Big\} \left[ i_3(\mathcal{T}_3, 1) - 2i_3(\mathcal{T}_3, v) + 2i_3(\mathcal{T}_4, v) \right] \\
& - \frac{e}{384} M^2 (e_u + e_d) \langle \bar{u}u \rangle \langle \bar{d}d \rangle (1-t)^2 \left[ 8i_3(\mathcal{S}, 1) + 4i_3(\mathcal{T}_1, 1) - 8i_3(\mathcal{T}_4, 1) - 8i_3(\mathcal{S}, v) \right. \\
& + 2\mathbb{A}'(u_0) + m_0^2 \chi \varphi'_\gamma(u_0) \Big] \\
& - \frac{e}{96} M^2 (1-t^2) \langle \bar{s}s \rangle \left\{ \left[ (e_u - e_s) \langle \bar{u}u \rangle + (e_d - e_s) \langle \bar{d}d \rangle \right] i_3(\mathcal{T}_1, 1) \right. \\
& + 6 \left[ (e_u + e_s) \langle \bar{u}u \rangle + (e_d + e_s) \langle \bar{d}d \rangle \right] i_3(\mathcal{S}, v) \Big\} \\
& + \frac{e}{192} M^2 f_{3\gamma} \left\{ m_u \langle \bar{u}u \rangle \left[ e_d(1+t)^2 - e_s(1+6t+t^2) \right] + m_d \langle \bar{d}d \rangle \left[ e_u(1+t)^2 - e_s(1+6t+t^2) \right] \right. \\
& - (e_u + e_d) m_s \langle \bar{s}s \rangle (1+6t+t^2) \Big\} i_4(\mathcal{A}, v) \\
& + \frac{e}{192} M^2 f_{3\gamma} \left\{ m_u \langle \bar{u}u \rangle \left[ e_d(1+t)^2 - e_s(3+2t+3t^2) \right] + m_d \langle \bar{d}d \rangle \left[ e_u(1+t)^2 - e_s(3+2t+3t^2) \right] \right. \\
& - (e_u + e_d) m_s \langle \bar{s}s \rangle (3+2t+3t^2) \Big\} i_4(\mathcal{V}, v) \\
& + \frac{e}{192} M^2 (1-t^2) \left[ (e_u - e_s) \langle \bar{u}u \rangle + (e_d - e_s) \langle \bar{d}d \rangle \right] \langle \bar{s}s \rangle \mathbb{A}'(u_0) \\
& + \frac{e}{48} M^2 f_{3\gamma} \langle \bar{u}u \rangle \left\{ e_d \left[ m_u(3+2t+3t^2) + 6m_s(1+t^2) \right] - e_s \left[ m_u(1+t)^2 + 2m_d(1-t)^2 \right] \right\} \psi^v(u_0) \\
& + \frac{e}{48} M^2 f_{3\gamma} \langle \bar{d}d \rangle \left\{ e_u \left[ m_d(3+2t+3t^2) + 6m_s(1-t^2) \right] - e_s \left[ 2m_u(1-t)^2 + m_d(1+t)^2 \right] \right\} \psi^v(u_0) \\
& + \frac{e}{48} M^2 f_{3\gamma} \langle \bar{s}s \rangle \left\{ e_u \left[ m_s(3+2t+3t^2) + 6m_d(1-t^2) \right] + e_d \left[ m_u(1-t)^2 + m_s(3+2t+3t^2) \right] \right\} \psi^v(u_0) \\
& + \frac{e}{2304\pi^2} M^2 e_u \langle g^2 G^2 \rangle \langle \bar{u}u \rangle (1-t) \left[ m_d(1-t) - m_s(1+t) \right] \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{1152} M^2 m_0^2 (e_u - 7e_s) \langle \bar{u}u \rangle \langle \bar{s}s \rangle (1-t^2) \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{2304\pi^2} M^2 e_d \langle g^2 G^2 \rangle \langle \bar{d}d \rangle (1-t) \left[ m_u(1-t) - m_s(1+t) \right] \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{1152} M^2 m_0^2 (e_d - 7e_s) \langle \bar{d}d \rangle \langle \bar{s}s \rangle (1-t^2) \chi \varphi'_\gamma(u_0) \\
& + \frac{e}{2304\pi^2} M^2 e_s (m_u + m_d) \langle g^2 G^2 \rangle \langle \bar{s}s \rangle (1-t^2) \chi \varphi'_\gamma(u_0) \\
& - \frac{e}{192} M^2 f_{3\gamma} \langle \bar{u}u \rangle \left\{ e_d \left[ m_u(3+2t+3t^2) + 6m_s(1-t^2) \right] + e_s \left[ m_u(1+t)^2 + 2m_d(1-t)^2 \right] \right\} \psi^{a'}(u_0) \\
& - \frac{e}{192} M^2 f_{3\gamma} \langle \bar{d}d \rangle \left\{ e_u \left[ m_d(3+2t+3t^2) + 6m_s(1-t^2) \right] + e_s \left[ 2m_u(1-t)^2 - m_d(1+t)^2 \right] \right\} \psi^{a'}(u_0) \\
& - \frac{e}{192} M^2 f_{3\gamma} \langle \bar{s}s \rangle \left\{ e_u \left[ m_s(3+2t+3t^2) + 6m_d(1-t^2) \right] + e_d \left[ m_u(1-t)^2 + m_s(3+2t+3t^2) \right] \right\} \psi^{a'}(u_0) \\
& - \frac{e}{768\pi^2} M^2 m_0^2 \langle \bar{u}u \rangle \left\{ e_d \left[ 3m_d(1-t)^2 - 18m_s(1-t^2) - 4m_u(1+t+t^2) \right] \right. \\
& - e_s \left[ 21m_s(1-t^2) + 2m_u(7+10t+7t^2) \right] \Big\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{e}{768\pi^2} M^2 m_0^2 \langle \bar{d}d \rangle \left\{ e_u \left[ 4m_d(1+t+t^2) + 18m_s(1-t^2) - 3m_u(1-t)^2 \right] \right. \\
& + e_s \left[ 2m_d(7+10t+7t^2) + 21m_s(1-t^2) \right] \left. \right\} \\
& + \frac{e}{768\pi^2} M^2 m_0^2 \langle \bar{s}s \rangle \left\{ e_u \left[ 18m_d(1-t^2) + 4m_s(1+t+t^2) + 15m_u(1-t)^2 \right] \right. \\
& + e_d \left[ 15m_d(1-t^2) + 4m_s(1+t+t^2) + 18m_u(1-t^2) \right] \left. \right\} \\
& + \frac{e}{1024\pi^4} M^8 \left[ (e_u + e_d)(7+6t+7t^2) - e_s(9+10t+9t^2) \right] \\
& - \frac{e}{128\pi^2} M^6 f_{3\gamma} \left[ 2(e_u + e_d) + e_s(1+6t+t^2) \right] i_4(\mathcal{A}, v) \\
& - \frac{e}{128\pi^2} M^6 f_{3\gamma} \left[ (e_u + e_d)(1+t^2) + e_s(3+2t+3t^2) \right] i_4(\mathcal{V}, v) \\
& + \frac{e}{64\pi^2} M^6 f_{3\gamma} \left[ (e_u + e_d)(3+2t+3t^2) - e_s((1+t)^2) \right] \psi^v(u_0) \\
& - \frac{e}{128\pi^2} M^6 (1-t) \left\{ e_u \langle \bar{u}u \rangle \left[ m_d(1-t) - m_s(1+t) \right] + e_d \langle \bar{d}d \rangle \left[ m_u(1-t) - m_s(1+t) \right] \right. \\
& + e_s \langle \bar{s}s \rangle (m_u + m_d)(1+t) \left. \right\} \chi \varphi'_\gamma(u_0) \\
& - \frac{e}{256\pi^2} M^6 f_{3\gamma} \left[ (e_u + e_d)(3+2t+3t^2) - e_s((1+t)^2) \right] \psi^{a'}(u_0) \\
& - \frac{e}{4608\pi^2} \frac{1}{M^2} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) - e_s m_d(1-t) \right] \right. \\
& + \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) - e_s m_u(1-t) \right] + 3\langle \bar{s}s \rangle (e_u m_d + e_d m_u)(1+t) \left. \right\} \left[ 3m_0^2 - 8\pi^2 f_{3\gamma} \psi^v(u_0) \right] \\
& - \frac{e}{2304} \frac{1}{M^2} f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) + e_s m_d(1-t) \right] \right. \\
& + \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) + e_s m_u(1-t) \right] + 3\langle \bar{s}s \rangle (e_u m_d + e_d m_u)(1+t) \left. \right\} \psi^{a'}(u_0) \\
& + \frac{e}{1152} \frac{1}{M^4} m_0^2 f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) - e_s m_d(1-t) \right] \right. \\
& + \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) - e_s m_u(1-t) \right] + 3\langle \bar{s}s \rangle (e_u m_d + e_d m_u)(1+t) \left. \right\} \left[ 3m_0^2 - 8\pi^2 f_{3\gamma} \psi^v(u_0) \right] \\
& - \frac{e}{4608} \frac{1}{M^4} m_0^2 f_{3\gamma} \langle g^2 G^2 \rangle (1-t) \left\{ \langle \bar{u}u \rangle \left[ 3e_d m_s(1+t) + e_s m_d(1-t) \right] \right. \\
& + \langle \bar{d}d \rangle \left[ 3e_u m_s(1+t) + e_s m_u(1-t) \right] + 3\langle \bar{s}s \rangle (e_u m_d + e_d m_u)(1+t) \left. \right\} \left[ 3m_0^2 - 8\pi^2 f_{3\gamma} \psi^{a'}(u_0) \right]
\end{aligned}$$

$$\begin{aligned}
& e^{m_{\Sigma_0}^2/M^2} \Pi_4(u, d, s) = \\
& \frac{e}{192} (1-t) \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s(1+t) - e_s m_u(1-t) \right] + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d m_s(1+t) - e_s m_d(1-t) \right] \right. \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u)(1-t^2) \left. \right\} \left[ i_3(\mathcal{S}, 1) - i_3(\mathcal{T}_4, 1) + 2i_3(\mathcal{T}_4, v) \right] \\
& + \frac{e}{192} (1-t) \left\{ \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u m_s(1+t) + e_s m_u(1-t) \right] + \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d m_s(1+t) + e_s m_d(1-t) \right] \right. \\
& - \langle \bar{u}u \rangle \langle \bar{d}d \rangle (e_u m_d + e_d m_u)(1-t^2) \left. \right\} \left[ i_3(\tilde{\mathcal{S}}, 1) + i_3(\mathcal{T}_2, 1) \right]
\end{aligned}$$

$$\begin{aligned}
& - \frac{e}{576} e_s (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle (1-t)^2 \left\{ 6 \left[ i_3(\mathcal{T}_3, 1) + i_3(\mathcal{T}_2, v) - 2i_3(\mathcal{T}_3, v) - 2\tilde{j}_1(h_\gamma) \right] \right. \\
& + \left. 3\mathbb{A}'(u_o) + m_0^2 \chi \varphi'_\gamma(u_0) \right\} \\
& - \frac{e}{96} (1-t^2) \left[ e_u m_s \langle \bar{u}u \rangle \langle \bar{s}s \rangle + e_d m_s \langle \bar{d}d \rangle \langle \bar{s}s \rangle - (e_u m_d + e_d m_u) \langle \bar{u}u \rangle \langle \bar{d}d \rangle \right] i_3(\mathcal{T}_2, v) \\
& - \frac{e}{48} \left\{ \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 3e_d m_s (1-t^2) + 2e_d m_u (3+2t+3t^2) - 2e_s m_u (1+t)^2 \right] \right. \\
& + \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 3e_u m_s (1-t^2) + 2e_u m_d (3+2t+3t^2) - 2e_s m_d (1+t)^2 \right] \\
& + \left. \langle \bar{u}u \rangle \langle \bar{d}d \rangle \left[ 3(e_u m_d + e_d m_u) (1-t^2) + 2(e_u + e_d) m_s (3+2t+3t^2) \right] \right\} \tilde{j}_1(h_\gamma) \\
& + \frac{e}{2304} m_0^2 f_{3\gamma} (1-t) \left\{ (1+t) \left[ \langle \bar{u}u \rangle (e_d - 7e_s) + \langle \bar{d}d \rangle (e_u - 7e_s) \right] - 3 \langle \bar{s}s \rangle (e_u + e_d) (1-t) \right\} \psi^{a'}(u_0) \\
& - \frac{e}{4608\pi^2} \langle g^2 G^2 \rangle f_{3\gamma} (1-t) \left\{ e_u \left[ m_d (1+t) - m_s (1-t) \right] + e_d \left[ m_u (1+t) - m_s (1-t) \right] \right. \\
& - \left. e_s (m_u + m_d) (1+t) \right\} \psi^{a'}(u_0) \\
& + \frac{e}{3072\pi^2} \langle g^2 G^2 \rangle f_{3\gamma} (1-t) \left( \gamma_E - \ln \frac{\Lambda^2}{M^2} \right) \left\{ e_u \left[ m_d (1+t) - m_s (1-t) \right] \right. \\
& + \left. e_d \left[ m_u (1+t) - m_s (1-t) \right] - e_s (m_u + m_d) (1+t) \right\} \psi^{a'}(u_0) \\
& + \frac{e}{48} M^2 e_s (m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) \langle \bar{s}s \rangle (1-t)^2 \chi \varphi'_\gamma(u_0) \\
& - \frac{e}{96} M^2 f_{3\gamma} (1-t) \left\{ e_u \left[ \langle \bar{d}d \rangle (1+t) - \langle \bar{s}s \rangle (1-t) \right] \right. \\
& + \left. e_d \left[ \langle \bar{u}u \rangle (1+t) - \langle \bar{s}s \rangle (1-t) \right] - e_s (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) (1+t) \right\} \psi^{a'}(u_0) \\
& - \frac{e}{288} \frac{1}{M^2} m_0^2 \left\{ m_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ 2e_u (5+4t+5t^2) + e_s (1+t)^2 \right] \right. \\
& + \left. m_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ 2e_d (5+4t+5t^2) + e_s (1+t)^2 \right] + 2(e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (5+4t+5t^2) \right\} \tilde{j}_1(h_\gamma) \\
& - \frac{e}{576} \frac{1}{M^4} \langle g^2 G^2 \rangle \left\{ m_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u (3+2t+3t^2) - e_s (1+t)^2 \right] \right. \\
& + \left. m_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d (3+2t+3t^2) - e_s (1+t)^2 \right] + (e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (3+2t+3t^2) \right\} \tilde{j}_1(h_\gamma) \\
& - \frac{e}{1152} \frac{1}{M^6} m_0^2 \langle g^2 G^2 \rangle \left\{ m_d \langle \bar{u}u \rangle \langle \bar{s}s \rangle \left[ e_u (3+2t+3t^2) - e_s (1+t)^2 \right] \right. \\
& + \left. m_u \langle \bar{d}d \rangle \langle \bar{s}s \rangle \left[ e_d (3+2t+3t^2) - e_s (1+t)^2 \right] + (e_u + e_d) m_s \langle \bar{u}u \rangle \langle \bar{d}d \rangle (3+2t+3t^2) \right\} \tilde{j}_1(h_\gamma) \\
& - \frac{e}{256\pi^2} M^4 e_s (1-t)^2 \left\{ \langle \bar{s}s \rangle \left[ 2i_3(\mathcal{S}, 1) - 2i_3(\tilde{\mathcal{S}}, 1) - 2i_3(\mathcal{T}_2, 1) + 4i_3(\mathcal{T}_3, 1) - 2i_3(\mathcal{T}_4, 1) + 4i_3(\mathcal{T}_2, v) \right. \right. \\
& - \left. \left. 8i_3(\mathcal{T}_3, v) + 4i_3(\mathcal{T}_4, v) - 4\tilde{j}_1(h_\gamma) + \mathbb{A}(u_0) \right] - f_{3\gamma} (m_u + m_d) \psi^{a'}(u_0) \right\} \\
& - \frac{3e}{64\pi^2} M^4 (e_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle) (1-t^2) \tilde{j}_1(h_\gamma) \\
& + \frac{e}{256\pi^2} M^4 f_{3\gamma} (1-t) \left\{ e_u \left[ m_d (1+t) - m_s (1-t) \right] + e_d \left[ m_u (1+t) - m_s (1-t) \right] \right. \\
& - \left. 2e_s (m_u + m_d) \right\} \psi^{a'}(u_0)
\end{aligned}$$

$$+ \frac{e}{192\pi^2} M^6 e_s \langle \bar{s}s \rangle (1-t)^2 \chi \varphi'_\gamma(u_0) \quad (1)$$

The functions  $i_n$ ,  $\tilde{i}_4$  and  $\tilde{\tilde{i}}_4$  are defined as

$$\begin{aligned} i_0(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) (k - u_0) \theta(k - u_0) , \\ i_1(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \theta(k - u_0) , \\ i_2(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta(k - u_0) , \\ i_3(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta'(k - u_0) , \\ i_4(\phi, f(v)) &= \int \mathcal{D}\alpha_i \int_0^1 dv \phi(\alpha_{\bar{q}}, \alpha_q, \alpha_g) f(v) \delta''(k - u_0) , \\ \tilde{j}_1(f(u)) &= \int_{u_0}^1 du f(u) , \\ \tilde{\tilde{j}}_2(f(u)) &= \int_{u_0}^1 du (u - u_0) f(u) , \end{aligned}$$

where

$$k = \alpha_q + \alpha_g \bar{v} , \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} , \quad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} .$$

# References

- [1] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
- [2] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1982).
- [3] B. Krusche, and S. Schadmand, Prog. Part. Nucl. Phys. **51**, 399 (2003).
- [4] M. Kortulla *et al.*, Phys. Rev. Lett. **89**, 272001 (2002); **61**, 147 (2008).
- [5] V. Punjabi *et al.*, Phys. Rev. C **71**, 055202 (2005).
- [6] N. Sharma, A. Martinez Torres, K. P. Khemchandani, H. Dahiya, Eur. Phys. J. A **49**, 11 (2013).
- [7] F. X. Lee, A. Alexandru, Proc. Sci., LATTICE2010, **(2010)** 148.
- [8] W. T. Chiang, and S. N. Yang, Nucl. Phys. **A723**, 205 (2003).
- [9] T. Hyodo, S. I. Nam, D. Jido, and A. Hosaka, arXiv:0305023.
- [10] I. M. Narodetski, and M. A. Trasov, arXiv:1311.2407.
- [11] V. M. Braun, arXiv:9801222.
- [12] V. Chung, H. G. Dosch, M. Kremer, and D. Scholl, Nucl. Phys. **B197**, 55 (1982).
- [13] T. M. Aliev, A. Özpineci, M. Savci, Phys. Rev. D **66**, 016002 (2002).
- [14] I. I. Balitsky, V. M. Braun, Nucl. Phys. **B311**, 541 (1989).
- [15] V. M. Braun, I. E. Filyanov, Z. Phys. C **48**, 239 (1990).
- [16] P. Ball, V. M. Braun, and N. Kivel, Nucl. Phys. **B649**, 263 (2003).
- [17] A. Özpineci, S. B. Yakovlev, and V. S. Zamiralov, Mod. Phys. Lett. A **20**, 243 (2005).
- [18] V. M. Belyaev, B. L. Ioffe, Sov. Phys. JETP **56**, 493 (1982).
- [19] K. G. Chetyrkin, A. Khodjamirian, A. A. Pirovarov, Phys. Lett. B **661**, 250 (2008).
- [20] J. Rohrwild, J. High Energy Phys. 09 (2007) 073.
- [21] C. A. Dominguez, N. F. Nasrallah, R. Rontisch, K. Schilcher, J. High Energy Phys. 05 (2008) 020.
- [22] J. Beringer *et al* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
- [23] D. Jido, A. Hosaka, J. Nacher, E. Oset, and A. Ramos, Phys. Rev. C **66**, 025203 (2002).